WORCESTER POLYTECHNIC INSTITUTE
TWENTY-EIGHTH ANNUAL INVITATIONAL MATH MEET
OCTOBER 20, 2015
TEAM EXAM QUESTION SHEET

DIRECTIONS: Please write your answers on the **TEAM ANSWER SHEET** provided. This part of the contest is 45 minutes. All 14 problems are counted equally. Calculators and other elecrtonics **MAY NOT** be used.

1. Find n so that $\frac{1^3+3^3+5^3+\ldots+(2n-1)^3}{2^3+4^3+6^3+\ldots+(2n)^3}=\frac{199}{242}$

2. **Simplify** $\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}$

3. **If $f(x) = x^4 + x^3 + x^2 + x + 1$ what is the remainder when $f(x^5)$ is divided by f(x) ?**

4. A football has been deflated so that each of it's principal dimensions is reduced to 90% of their original value. What percentage of the original volume is the new volume?

5. Simplify $\mathbf{13^{8231} \bmod 17}$

6. Consider the complex number **3 – 3i.** Convert it to polar form.

7. Suppose a sequence is explicitly defined by

$$X_k=\frac{\left(\frac{1+\sqrt{5}}{2}\right)^k-\left(\frac{1-\sqrt{5}}{2}\right)^k}{\sqrt{5}}$$

find a simplified form for X_{11}

8. Consider the following two-player game: A coin with probability .7 of heads is to be tossed. This probability is known to both participants. Each player can choose a sequence of length three of heads and tails, and the player whose sequence appears first in a repeated coin toss wins. Player 1 chooses the sequence HHH. Player 2 is to choose a different three outcome sequence. (For example if player 2 chooses HTT and the first five tosses are HTHHH, player 1 wins.) Player 2 chooses a sequence to maximize his/her probability of winning. What is player 2's probability of winning with that sequence?

9. If $\log_8(\log_3(\log_2(x))) = 0$ then $x^{1/3}$ is equal to what ?

10. In Z_{19}, the field of integers mod **19**, the number **8** has 3 cube roots. One of them is 2. What are the other two?

11. What is

$$\sum_{r=0}^{24}\sum_{s=1}^{40}\left(\frac{1}{s}-\frac{1}{s+1}\right) \quad ?$$

12. A motorist wishes to have a 50-50 mix of water and antifreeze. He is given 1.5 gallons of a mix and told it is only 25% water. How much water, in gallons, should he add so that the resulting mix is 50-50?

13. Books have 10 digit **ISBN**s. The last digit is the *check digit*. If the general form is

$$a_1 a_2 a_3 \ldots a_{10}$$

then error checking software forms the linear combination

$$(10\,a_1 + 9a_2 + 8a_3 + \ldots + a_{10}) \bmod 11$$

which is supposed to come out to **0** for a correct code.

Suppose one digit of an ISBN has been smudged and looks like

0-716?-2841-9

Figure out the missing digit.

14. Consider the binary number 111111111111111_2. Is it a prime number?